Proceedings
The 2nd International Multidisciplinary Conference 2016
November 15th, 2016, Universitas Muhammadiyah Jakarta, Indonesia
Krisnamurti et al., Flexural Analysis of HPFRC Plates Subjected by Monotonic Loading using 2-D Isoparametric Element and Mindlin Plate Element: 354-361
ISBN 978-602-17688-9-1

# FLEXURAL ANALYSIS OF HPFRC PLATES SUBJECTED BY MONOTONIC LOADING USING 2-D ISOPARAMETRIC ELEMENT AND MINDLIN PLATE ELEMENT

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## Abstract

*Finite element analysis has been widely used to describe the behavior of High-Performance Fiber Reinforced-Concrete (HPFRC) structural elements under a variety of loadings. This study deals with the finite element analysis of the monotonic flexural behavior of HPFRC plates. Following the assumptions of the development plate theory, the finite element idealization of HPFRC plates can be accomplished by 2-dimensional plane stress isoparametric element or by Mindlin plate element. The aim of this research is to develop methods of finite element modeling of flexural behavior which occurred at HPFRC plates subjected to monotonic loading using two different methods of element idealization which accordance to the laboratory testing result. The analysis is done by decreasing the stiffness of plate elements gradually in accordance with the development of maximum stress in the element due to workload. Correlation studies between analytical and experimental results are conducted with the objective to establish the validity of the proposed models and identify the significance of various effects on the response of HPFRC plates element. Flexural analysis conducted on plate specimens with 600 mm span length, 300 mm width, and 50 mm thickness. HPFRC compressive strength is 93.045 MPa, and splitting tensile strength is 6.018 MPa. Test performed with four points bending pattern at a distance of 1/3 span length. Comparison results of analysis and laboratory test can be concluded that the flexural behavior of plate HPFRC can be described more satisfactorily through finite element analysis using 2D-isoparametric plate element with minimum element size. However, if a significant number of elements is added in the calculation method of Mindlin plate, then the overall result of the analysis will be closer to the experimental results.*

***Keywords:*** *HPFRC plate, Mindlin plate element, 2-D isoparametric element, four points bending.*

## INTRODUCTION

High-Performance Fiber-Reinforced Concrete (HPFRC) as high strength concrete become one of the most important building materials and widely used in many types of engineering structures. HPFRC as structural material must satisfy the conditions that the structure must be: (1) strong and safe. (2) stiff and appear unblemished. (3) economical. (Kwak & Filippou, 1990). HPFRC have a greater role as beams or plates. With the application of HPFRC technology, self-weight of bridge structure becomes lighter, but traffic load remains underserved as planned.

Plates are flat elements that have small thickness compared to surface area. Plates tend to behave as the bending elements when receiving load perpendicular to the plane surface. Load variations which acting on the bridge plate depend on the function, geometry, span length and construction methods of the bridge. If plate deflections are small in comparison with thickness, approximate theory of plate bending by lateral loads can be developed by assumptions: 1) There is no deformation in the middle plane of the plate. 2) Points of the plate lying initially on a normal-to-the-middle plane of the plate remain on the normal-to-the-middle surface of the plate after bending. 3) The normal stress in the direction transverse to the plate can be disregarded. All stress components can be expressed by deflection of the plate, which is a function of the two coordinates in the plane of the plate (Timoshenko & Woinowsky-Krieger, 1987).

By following assumptions of the plate theory, finite element idealization of HPFRC plates can be accomplished with two-dimensional isoparametric plane stress element or by Mindlin plate element. The aim of this research is to develop methods of finite element modeling of flexural behavior which occurred at HPFRC plates subjected to monotonic loading using two different methods of element idealization which accordance with the experiment result.

## Finite Element Modeling

In the design of HPFRC structure, it is expected that service loads will produce tensile stresses that exceed the strength of plain concrete and result in cracking. To assure that cracks will not impair the serviceability of a structure, the designer first proportions the members for adequate safety and then investigates the suitability of crack widths and deflections under service loads. In a simply-supported flexural member, one part of the cross section must carry the compression force to provide the internal bending moment to resist the applied external load. Part of the concrete has to be in compression at all times, thereby eliminating the possibility of a crack through the full depth (Carino & Clifton, 1995).

This study deals with the finite element analysis of the monotonic flexural behavior of HPFRC plates. A finite element model that takes into account the detailed behavior of HPFRC materials and their interaction is very useful since it provides a reliable analysis of the non-linear behavior of HPFRC bending element (Oliveira, Ramalho, & Correa, 2008). Every nonlinear analysis algorithm consists of four basic steps: 1) the formation of the current stiffness matrix, 2) the solution of the equilibrium equations for the displacement increments, 3) the state determination of all elements in the model, and 4) the convergence check. These steps are almost same for the plane stress and the plate bending problem (Kwak & Filippou, 1990).

In this study, flexural analysis conducted on HPFRC plate specimens with 600 mm pure span length, 300 mm width, and 50 mm thickness. The compressive strength of HPFRC is 93.045 MPa, and splitting tensile strength is 6.018 MPa. Test performed with four points bending pattern at a load distance of 1/3 span length. The geometrical properties of HPFRC plate are shown in Figure 1.

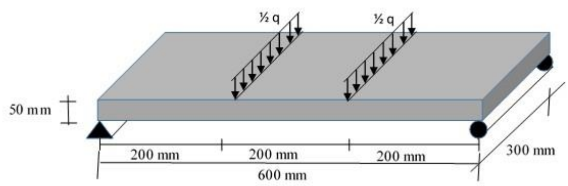


Figure 1. Geometrical properties of HPFRC plate

### *Modeling of 2-D Isoparametric Element*

When the HPFRC plate has cracked, the behavior of HPFRC material is approached by the anisotropic theory. Cracks that occur in the HPFRC plate will reduce the effective height of the cross section and further decreases the section's moment of inertia (I) (Tudjono, Pamungkas, & Lie, 2014). In order to analyze the deflection of the HPFRC plate using 2D-isoparametric element, a typical HPFRC plate finite element is divided into imaginary HPFRC mesh layers. In this study, it is about 12 element in the x-direction. The variation in material properties through the thickness is discretized by dividing the plate into layers, within each of which the material properties are constant (Dotreppe, Schnobrich, & Pecknold, 1973). They present a homogeneous structure through the thickness of the layer. It is assumed that the displacement area of the member is continuous and there are no gaps between layers (Sezer & Tekin, 2011). The cross section of HPFRC plates with a dimension of 600 mm span length and 50 mm depth is divided into a 2D-isoparametric element with a size of the 2x12 element, 4x12 element, and 5x12 element. This treatment is to show the influence of number element to the accuracy of the analysis. Figure 2 shown the cross section of HPFRC plate divided into the 5x12 element. The

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workload is act only in the y-direction, ½ P at the joint of element 52 - 53 and ½ P at the joint of element 56 and 57.

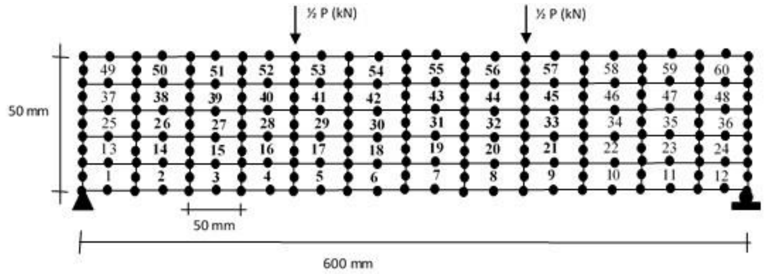



Figure 2. Modeling of HPFRC plates by 2D-isoparametric element

The external loading is applied in increments. The analysis is done by decreasing the stiffness of plate elements gradually in accordance with the development of maximum stress in the element due to workload as shown in table 1. At the beginning of each load increment, the structure tangent stiffness matrix is updated to reflect any changes in material properties which have taken place. Inelastic action within the load increment is taken into account by the "initial stress" method (Zienkiewicz, O.C. & Taylor, R.L., 2000), in which pseudo-loads reflecting the inelastic behavior are iteratively redistributed through the structure using the tangent stiffness computed at the beginning of the increment (Dotreppe, Schnobrich, & Pecknold, 1973).

Table 1. Decreasing effective stiffness (δE) of plate element

| No | Maximum Stress (MPa) | δE (MPa) |
|---|---|---|
| 1 | 0 | 38925 |
| 2 | 1.018 | 32525 |
| 3 | 2.018 | 26125 |
| 4 | 3.018 | 19725 |
| 5 | 4.018 | 13325 |
| 6 | 5.018 | 6925 |
| 7 | 6.018 | 1525 |

***Modeling of Mindlin Plate Element***

In this modeling, HPFRC plate bending element is developed based on the Mindlin plate theory and the layer concept. HPFRC plate is analyzed as a quadrilateral element with a size of the 3x3 element, 6x6 element, and 12x12 element.

Table 2. Decreasing effective depth of plate element

| No. | Maximum stress (MPa) | Effective plate depth (mm) |
|---|---|---|
| 1 | 0 | 50 |
| 2 | 1,018 | 45 |
| 3 | 2,018 | 40 |
| 4 | 3,018 | 35 |
| 5 | 4,018 | 30 |
| 6 | 5,018 | 25 |
| 7 | 6,018 | 20 |

ISBN 978-602-17688-9-1

Figure 3 shown the discretized of the surface of the HPFRC plate into 12x12 quadrilateral element. The element is capable of modeling the gradual propagation of cracks through the depth of the slab. For out-of-plane loading, the assumptions of the Mindlin plate bending theory are: (1) transverse displacements are small relative to the plate thickness, (2) the stress normal to the mid-surface of the plate is negligible, and (3) normals to the mid-surface of the slab before deformation remain straight, but not necessarily normal to the mid-surface after deformation (Kwak & Filippou, 1990). The analysis is done by decreasing effective depth of plate elements gradually in accordance with the development of maximum stress in the element due to workload as shown in table 2.

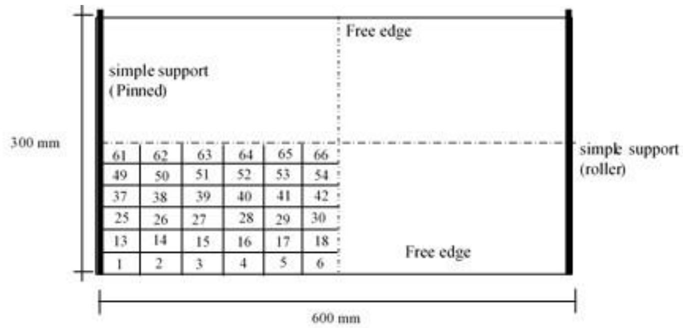



Figure 3. Modeling of HPFRC plate by Mindlin plate elements with size of 12x12

## EXPERIMENTAL METHODOLOGY

Bending test performed on a simple supported HPFRC plate specimens. Work loading with a distance of 1/3 span plate is used in the four points bending of the plate. In this model, the loads applied were increased simultaneously by displacement control at upper middle points of the plate. Three pieces of HPRFC plate specimens are made for the test. Every mixing process, one HPFRC cylinder with diameter 100 mm x 200 mm also made for testing compressive strength in accordance with the results of research (Graybeal & Davis, 2008) and (Habel, Viviani, Denarie, & Bruhwiler, 2006) for high-performance concrete, and one cylinder diameter 150 mm x 300 mm for testing the tensile strength as shown in Fig. 4. The average cylinder compressive strength is about 93.045 MPa and tensile strength dimpled 6.018 MPa. The average modulus of elasticity is about 38925 MPa. Modulus of elasticity HPFRC calculated by the equation:

$$E = 3320\ \sqrt{fc'} + 6900\ MPa \quad (1)$$

according to the ACI 363R equation in (Graybeal B. A., 2007) and (Noguchi, Tomosawa, Nemati, Chiaia, & Fantili, 2009). E and fc' in MPa.

After several mix design testing of HPFRC, the material constituent composition in this study consist of weight: 1 cement: 0.1 silica fume: 0.1 Silica powder: 0.7 sand and gravel: 0.4 water: 0.22 superplasticizer: 0.15 tie wire. Fine sand taken from Mount Semeru and gravel crushed stone with a diameter less than 10 mm are used as HPFRC aggregate material. Master Glenium SKY 8851 is used as superplasticizer. Tie wire used as a fiber because has a measured tensile strength of testing according to the results of research (Aulia & Rinaldi, 2015) and (Widodo, 2012).

Figure 4. Compressive and splitting test of HPFRC cylinder

ISBN 978-602-17688-9-1

Figure 5. Casting HPFRC plate and Preparing loading test

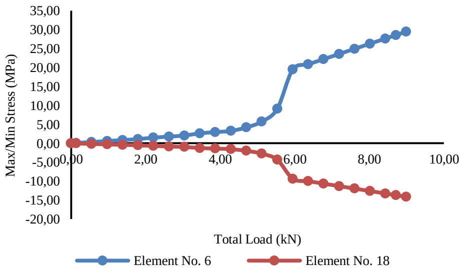
Figure 6. Setting proving ring and dial gauge, bending pattern of HPFRC plate specimens

## RESULTS AND DISCUSSION

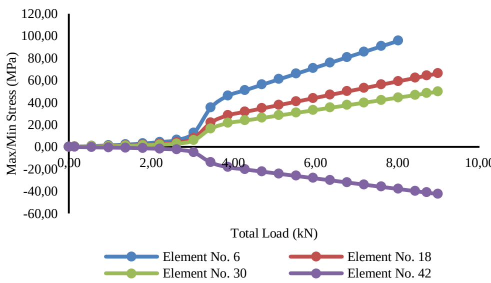
The finite element adopted in this study has a layered cross section. For the analysis with 2D-isoparametric elements, plate cross section is discretized into 2x12 elements, 4x12 elements, and 5x12 elements. Then for the analysis with Mindlin plate element, plate surface is discretized into 3x3 elements, 6x6 elements, and 12x12 elements. This treatment is done to shown the accuracy of the results analysis by a number of the element. The relationship of strength and load that occurs as shown in Figure 7.

The non-linear behavior of HPFRC plates shown in the relationship of max/min stress and total load in the figure 7a, 7b, and 7c. Analyses were performed using 2D-isoparametric elements. Principle stress will increase accordance with the number of elements which used in the analysis. With the total load up to 8.99 kN, the max/min stress results with the 2D-isoparametric element are 29.47 MPa and -14.12 MPa for analysis with 2x12 elements and increase to 95.54 MPa and - 42.46 MPa for analysis with 4x12 elements and eventually became 95.01 MPa and - 60.57 MPa for analysis with 5x12 elements.

Non-linear behavior also is shown in Figure 7d, that is a relationship of max/min stress and total load when analysis performed using Mindlin plate elements. For the analysis with the number of elements 3x3, the stress value is 1.88 MPa and -0.53 MPa. Furthermore, for a number of elements 6x6 obtained the stress value of 15.46 MPa and - 4.61 MPa. Eventually,when the analysis carried out on the number of 12x12 elements, it is obtained stress value of 94.20 MPa and - 26.59 MPa. The maximum total load is 8.99 kN.




*a. 2D-isoparametric FEM with 2x12 element*




*b. 2D-isoparametric FEM with 4x12 element*

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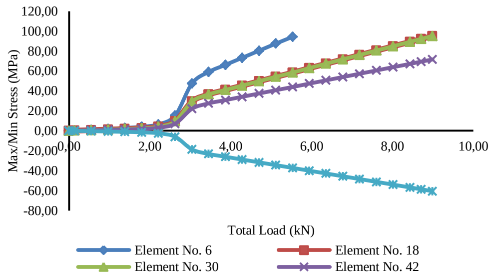
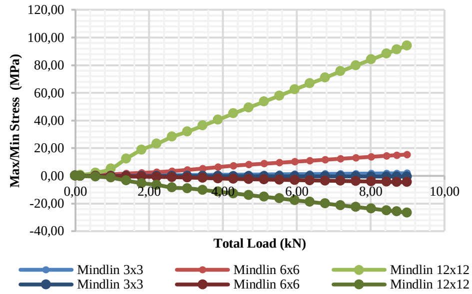


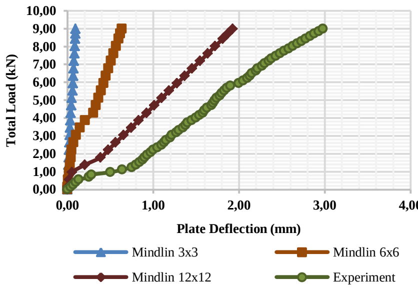

c. *2D-isoparametric FEM with 5x12 element*



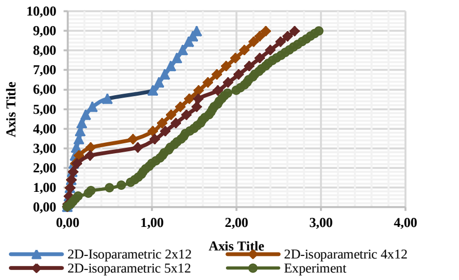

d. *Mindlin plate element with 3x3, 6x6, and 12x12 element*

Figure 7. Max/Min Stress–Load relationship of HPFRC plates

The relationship between total load and displacement shown in Figure 8. The results of the analysis with Mindlin plate element shown that a number of elements used to perform finite element analysis, will provide HPFRC plate non-linear behavior better. Calculations with a 3x3 element only produce deflection 0.10 mm. While the 6x6 element produces 0.64 mm deflection and 12x12 elements yield 1.93 mm deflection. The analysis with 2-D isoparametric element also shows that a number of elements will improve the accuracy of calculation result data. For 2x12 elements, it is obtained deflection of 1.53 mm, then with 4x12 elements, the result is 2.53 mm and eventually with 5x12 elements, obtained a yield of 2.69 mm. It is close to deflection value 2.98 mm of experimental results.




a. *Mindlin plate element with 3x3, 6x6, 12x12 element and experiment result of HPFRC plate*




b. *A 2d_isoparametric element with 2x12, 4x12, 5x12 element, and experiment result of HPFRC plate.*

Figure 8. Load-Deflection relationship of HPFRC plates

Correlation between analytical and experimental results are conducted with the objective to establish the validity of the proposed models and identify the significance of various effects on the response of HPFRC element plates. From Figure 9 it can be shown that in the initial stage, the analysis with Mindlin plate element could show the non-linear behavior of the plates HPFRC which is closer to the experimental results. However, after the cracks occurred and the stress exceeds HPFRC tensile stress, results of the analysis with 2D-isoparametric elements is more close to experiment results. This suggests that to know the magnitude of the deflection of the end of the plates HPFRC test will be faster if using 2D-isoparametric element method.

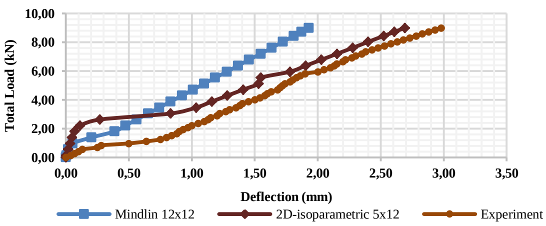



Figure 9. Comparison Load-Deflection of the 2D-isoparametric element, Mindlin plate element, and the experiment.

## CONCLUSIONS

From the analysis of HPFRC plate using 2D-isoparametric element and Mindlin plate element can be shown that number of elements used to perform flexural behavior analysis of HPFRC plate greatly affect the accuracy of the results when compared with experimental testing. Analysis using the Mindlin plate element has the advantage of the proximity of deflection value in the early stage of experimental results, but after tensile stress exceeds the tensile capacity of HPFRC plate, the deflection began to move away. It required the addition number of elements to achieve results that actually approached the experimental test results. Analysis using 2D-isoparametric element showed that before HPFRC tensile strength is exceeded, deflection value is much higher than the actual deflection. However, with a number of the 5x12 element only, the results of the analysis is nearing the end value deflection plate HPFRC when collapsed.

Comparison results of analysis and laboratory test can be concluded that the flexural behavior of plate HPFRC can be described more satisfactorily through finite element analysis using 2D-isoparametric plate element with minimum element size. However, if a significant number of elements is added in the calculation method of Mindlin plate, then the overall result of the analysis will be closer to the experimental results.

## Acknowledgement

This research can be established with a grant program of BPPDN University of Brawijaya Malang Indonesia for programming process and Grant Program of “ the Hibah Bersaing” University of Jember for a laboratory experiment.

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