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STUDY ON FREE VIBRATION OF SKEW PLATE USING FINITE ELEMENT METHOD

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ABSTRACT

A numerical procedure is presented for free vibration analysis of skew plates using the finite element method. The finite element method is a tool used by many researchers to analyze plates element and gives good results. In this research, the natural frequency of the plate is obtained through free vibration analysis, the analysis utilizes skew plate elements with several skew angles and varying length to thickness ratios. The reference solution in the literature will be used as a comparison to obtain the convergence behavior of the plate element.

Keywords: *Free Vibration, Natural Frequency, Skew plate, Finite element.*

1. INTRODUCTION

Plate elements are structures that are widely used in construction, along with the development of increasingly complex structural shapes requiring a tool to analyze. Finite element method arise a solution to solve plate problem, this method works by discreatizating element in tho small pieces called mesh, connected at each node. The ability of a finite element plate to discretize an element determines the analysis results. The phenomenon of shear locking will occur if the finite element plate element cannot decretize the element properly, especially when the ratio of length to thickness increases. (L/h) [1].

The dependence of a plate element on its thickness cause the analysis result inaccurate. Overcoming the shear locking on plate analysis, many approaches have been proposed by researcher, MITC3 (3-node triangular mixed interpolation of tensorial components) developed by Lee & Bathe [2] is one of plate finite element that have given good result in plate problem particularly on quadrilateral elements [3],[4].

The MITC3 element's development and research can be found in [1],[2],[5], and [6]. The finite element method has been used to conduct research on free vibration analysis [7]-[8]. In this study, we carrie out a free vibration analysis using MITC3 plate finite element in the case of free vibration analysis, several skew angle of skew plate, and two types of length to thickness ratios (L/h). Comparing reference solution in the literature and the analysis result will be obtained the convergence behavior of MITC3 plate element.

2. REISSNER-MINDLIN PLATE THEORY

Strain-Displacement Equation

The displacement field can be defined as follows using Reissner-Mindlin plate first-order shear deformation plate theory [9].

$$\begin{aligned} u(x,y,z) &= u(x,y) + z\beta_x(x,y) \\ v(x,y,z) &= v(x,y) + z\beta_y(x,y) \\ w(x,y,z) &= w(x,y) \end{aligned} \tag{1}$$

where u, v, and w are displacements of the mid-plane of the plate, βx and βy represent the rotations of the transverse normal about the x and y axes, respectively.
The linear strains are given by:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix} = z\{\chi\} \tag{2}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \begin{Bmatrix} \beta_x + w_{,x} \\ \beta_y + w_{,y} \end{Bmatrix} = \{\gamma\} \tag{3}$$

The notation of $\beta_{x,x}$ and $\beta_{y,y}$ state the first derivatives with respect to x and y respectively of β_x, $w_{,x}$ and $w_{,y}$ are the first derivatives with respect to x and y respectively of vertical displacement w.

Constitutive Equations

According to hook law's the stress in the plane can be declared as:

$$\{\sigma\} = [\mathrm{E}]\{\varepsilon\} \tag{4}$$

The shear stress is as follows

$$\{\tau\} = [\mathrm{G}]\{\gamma\} \tag{5}$$

The constitutive equations can be simplified as:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{(1-v)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{6}$$

$$\begin{Bmatrix} \tau_x \\ \tau_y \end{Bmatrix} = \frac{E}{2(1+v)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

Where the matrix of material is:

$$[\mathrm{E}] = \frac{E}{1-v^2} \begin{bmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{(1-v)}{2} \end{bmatrix} \tag{7}$$

$$[\mathrm{G}] = \frac{k\ E}{2(1+v)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The notation of E declares the modulus of elasticity, v is Poisson's ratio, and k= 5/6 is the shear correction factor.

3. THE FORMULATION OF MITC3 PLATE ELEMENT

MITC3 is the element proposed by Lee and Bathe, this element is based on the concept of "mixed interpolation of tensorial components" (MITC) by Dvorkin and Bathe [4] which uses the tying points to get the shear strain matrix.
MITC3 element has 3 nodes with 3 degrees of freedom for each node, namely: w_i (translation in the z-direction) β_{xi} (rotation in the z-x plane), and β_{yi} (rotation in the z-y plane).

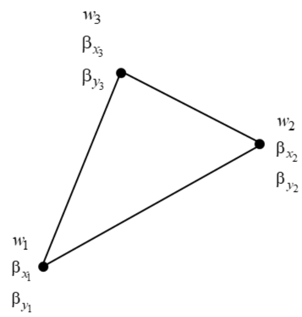


Figure 1. MITC3 elements with 3 nodes and 3 dof per node.

Independent rotation field β_x and β_y and displacement w are declared as:

$$
\begin{aligned}
w &= \sum_{i=1}^{3} = N_i w_i \\
\beta_x &= \sum_{i=1}^{3} = N_i \beta_{x_i} \\
\beta_y &= \sum_{i=1}^{3} = N_i \beta_{y_i}
\end{aligned}
\tag{8}
$$

Where N_i is the linear shape function at node-i.

$$
\begin{aligned}
N_1 &= 1 - \xi - \eta \\
N_2 &= \xi \\
N_3 &= \eta
\end{aligned}
\tag{9}
$$

The Bending Strain for MITC3

The relationship between nodal variables and curvature $\{\chi\}$ is:

$$
\{\chi\} = [B_b]\{u_n\}
$$

$$
[B_b] = \begin{bmatrix} 0 & N_{i,x} & 0 \\ \ldots 0 & 0 & N_{i,y} \ldots \\ 0 & N_{i,y} & N_{i,x} \end{bmatrix}_{i=1,2,3}
\tag{10}
$$

The Shear Strain for MITC3

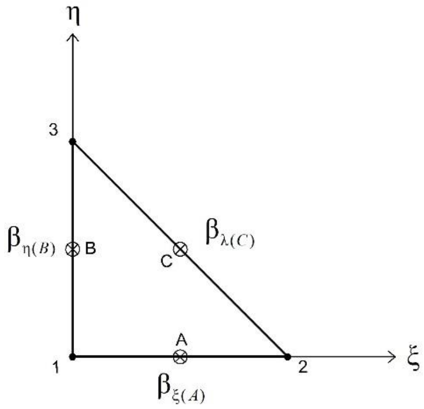


Figure 2. Tying point

The tying point is chosen in the mid-points of sides 1-2, 1-3, and 2-3. Distribusi β_ξ is assumed constant ξ and β_η is assumed constant along η.

$$
\begin{aligned}
\beta_\xi &= a_1 + a_2\eta \\
\beta_\eta &= b_1 + b_2\xi \\
\beta_\lambda &= \frac{1}{\sqrt{2}}(\beta_\xi - \beta_\eta)
\end{aligned}
\tag{11}
$$

The value of β_ξ and β_η, at the tying points, is the average of the values for each corner of the side, then:

$$
\begin{aligned}
\beta_{\xi(A)} &= \frac{1}{2}\left(\beta_{\xi_1} + \beta_{\xi_2}\right) \\
\beta_{\eta(B)} &= \frac{1}{2}\left(\beta_{\eta_1} + \beta_{\eta_3}\right) \\
\beta_{\xi(C)} &= \frac{1}{2}\left(\beta_{\xi_2} + \beta_{\xi_3}\right) \\
\beta_{\eta(B)} &= \frac{1}{2}\left(\beta_{\eta 3} + \beta_{\eta_3}\right)
\end{aligned}
\tag{12}
$$

Hence, the shear strain matrix as below;

$$
[B_s] = [j]\left[B_{s_\xi}\right] = \left[\left[B_{s_1}\right] \; \left[B_{s_2}\right] \; \left[B_{s_3}\right]\right]
\tag{13}
$$

The Shear Strain for MITC3

The total energy due to bending and shear can be stated as:

$$
\Pi_{\text{int}} = \Pi_{\text{int}}^{b} + \Pi_{\text{int}}^{s}
\tag{14}
$$

Where Π_{int}, Π_{int}^{b}, and Π_{int}^{s} are internal, bending, and shear energy respectively.

The total stiffness due to bending, and shear can be expressed as:

$$
[k] = [k_b] + [k_s]
\tag{15}
$$

Where

$$
\begin{aligned}
[k_b] &= \int_A [B_b]^T [H_b][B_b] dA \\
[k_s] &= \int_A [B_s]^T [H_s][B_s] dA
\end{aligned}
\tag{16}
$$

4. FREE VIBRATION ANALYSIS

In free vibration analysis, the thing to be achieved is to get the natural frequency value ω due to a given mass and get a mode shape *d*. To get these two variables, it can be solved through the eigenproblem [11]:

$$\left([k]-\omega_n^{2}[m]\right)\{d\}=\{0\} \tag{17}$$

Where,

$[k]$ = Structural Stiffness Matrix

ω_n = Natural frequency

$[m]$ = Mass matrix

$\{d\}$ = mode shape

Mass Matrix

The external energy equation due to free vibrations on the plate is

$$\Pi_{ext}=\langle u_n\rangle[m]\{u_n\} \tag{18}$$

The nodal displacement vectors

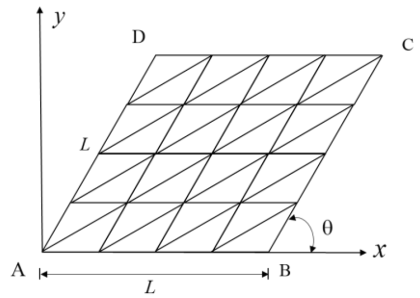
$$\langle u_n\rangle=\langle w_1 \quad \beta_{x1} \quad \beta_{y1} \quad w_2 \quad \beta_{x2} \quad \beta_{y2} \quad w_3 \quad \beta_{x3} \quad \beta_{y3}\rangle \tag{19}$$

The mass matrix in the free vibration analysis of the plate can be expressed as

$$[m]=[m_{\mathrm{w}}]+\left[m_{\beta_{\mathrm{x}}}\right]+\left[m_{\beta_{\mathrm{y}}}\right] \tag{20}$$

5. NUMERICAL ANALYSIS

In this paper, numerical analysis was carried out on a fully modeled CFCF skew plate. the element was restrained (w=0, βx=0, and βy=0) on AB and CD sides, and free on the AD and BC sides. Analyzing skew plate elements utilize two types of length to thickness ratio of L/h = 5 and L/h = 1000, and four types of skew angle of 30°, 45°, 60°, and 75°. To examine the convergence behavior of the MITC3 element on free vibration case, several variations in mesh size are used of (N × N × 2) of 4 × 4 × 2, 8 × 8 × 2, 16 × 16 × 2, 3 × 32 × 2, 64 × 64 × 2, 128 × 128 × 2. The convergence behavior of the skew plate will be ratified using the reference of natural frequency which is delivered by Woo et al. $\underline{\omega}=\left(\omega a^2/\pi^2\right)\sqrt{\rho h/D}$ [18].



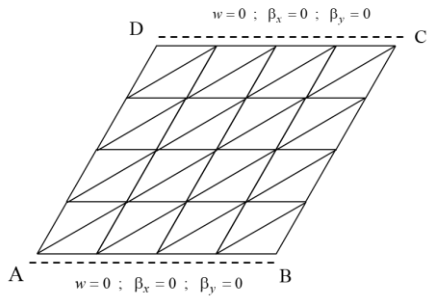


Figure 3. The modeling of the skew plate with CFCF boundary conditions.

Table 1. Fundamental frequency parameter for SCSC skew plate when L/h=5.

N × N × 2	***Mode number***	***Skew angle θ***	
		***θ* =30°**	***θ* = 45°**
4 × 4 × 2	1	4.9793	3.2465
8 × 8 × 2	1	4.3102	2.8299
16 × 16 × 2	1	3.9403	2.6624
32 × 32 × 2	1	3.7215	2.5983
64 × 64 × 2	1	3.6141	2.5761
128×128×2	1	3.5748	2.5684
Woo et al. [7]	1	3.5597	2.5659

Table 2. Fundamental frequency parameter for SCSC skew plate when L/h=5.

N × N × 2	***Mode number***	***Skew angle θ***	
		θ = 60˚	***θ = 75˚***
4 × 4 × 2	1	2.4827	2.1339
8 × 8 × 2	1	2.2013	1.9195
16 × 16 × 2	1	2.1156	1.8644
32 × 32 × 2	1	2.0891	1.8493
64 × 64 × 2	1	2.0809	1.8451
128×128×2	1	2.0783	1.8439
Woo et al. [8]	1	2.0788	1.8465

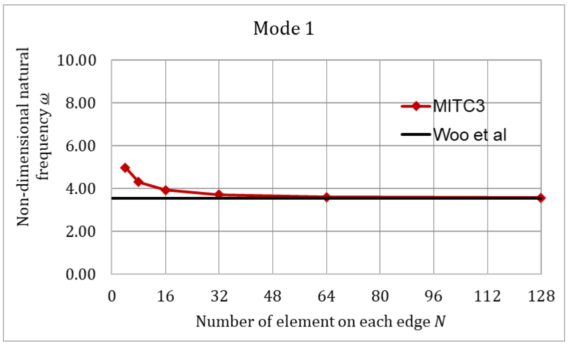


Figure 4. The natural frequency of CFCF skew plate (L/h = 5) with skew angle θ = 30.

The analysis in Table 1-2 and Fig. 4, indicated that MITC3 element convergence to reference solution in the case of the thick plate element (L/h=5). Skew plate on mesh 4 × 4 × 2 to mesh 128 × 128 × 2, the difference in mode 1 skew angle θ = 30˚ to reference solutions is 0.424% - 39.879%, θ = 45˚ is 0.096% - 26.526%, θ = 60˚ is 0.023% - 19.430%, and θ = 75˚ is 0.141% - 15.562%.

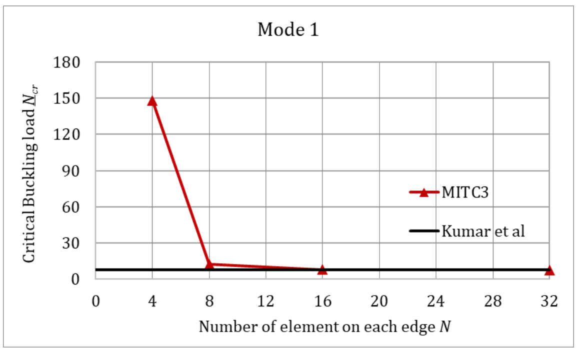


Figure 4. The natural frequency of CFCF skew plate (L/h = 5) with skew angle θ = 45.

Fig. 5 exhibits that a plate with skew angle θ = 45˚ has a smaller natural frequency than θ = 30˚, in fine mesh, and mode 1, the difference is 1.000. In the case of θ = 45˚ MITC3 elements converge to the reference solution.

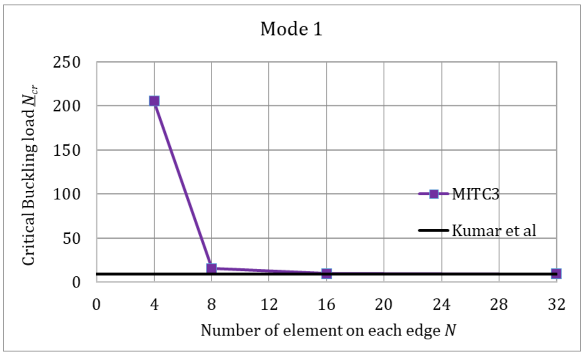


Figure 6. The natural frequency of CFCF skew plate (L/h = 5) with skew angle θ = 60.

Fig. 6 represents that MITC3 convergence to reference solution, in fine mesh, skew plate θ = 45˚ has a bigger natural frequency than θ = 60˚. The difference of each other is 0.4900.

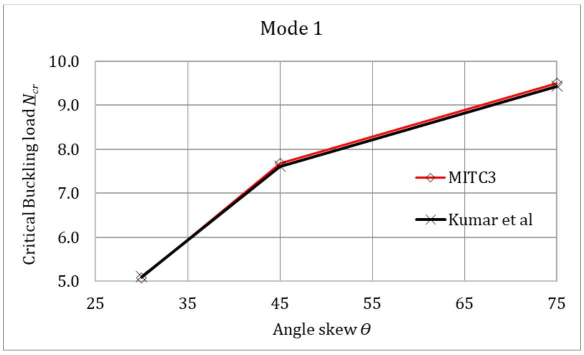


Figure 5. The natural frequency of CFCF skew plate (L/h = 5) with skew angle θ = 75.

Fig. 5 denotes that skew plate θ = 75˚ has a natural frequency that is the smallest of any skew plate in this research. In addition, From the several skew plate, it is obtained that θ = 30˚ has the largest difference to the reference solution than θ = 45˚, θ = 60˚, and θ = 75˚. In the case of the thick plate (L/h=5) obtained that increasing the skew angle of the plates θ caused a decrease in the natural frequency values.

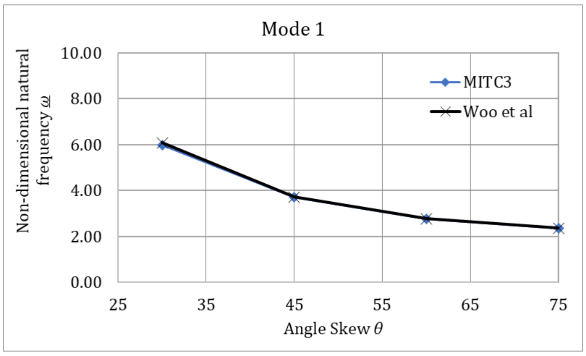
Table 3. The first five non-dimensional natural frequency parameters of CFCF skew plate (L/h=1000).

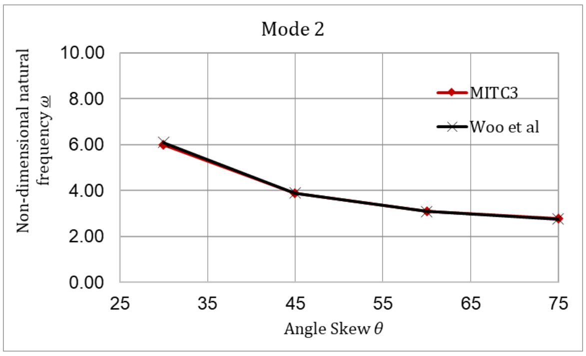
N × N × 2	***Mode number***	***Skew angle θ***	
		***θ* = 30°**	***θ* = 45°**
128 × 128 × 2	1	5.9785	6.0748
	2	5.9949	6.1030
	3	9.9372	9.8642
	4	12.2506	12.0750
	5	15.5102	15.7160
Woo et al. [8]	1	3.7094	3.7149
	2	3.8904	3.8948
	3	6.2970	6.2744
	4	8.9132	8.8512
	5	10.5553	10.6030

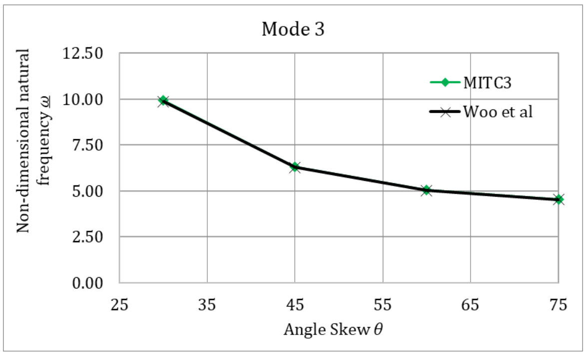
Table 3. The first five non-dimensional natural frequency parameters of CFCF skew plate (L/h=1000).

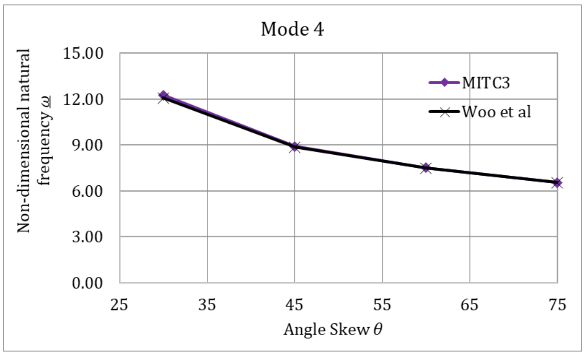
N × N × 2	***Mode number***	***Skew angle θ***	
		***θ* =60°**	***θ* = 75°**
128 × 128 × 2	1	2.7783	2.7767
	2	3.1044	3.0890
	3	5.0351	5.0186
	4	7.5166	7.4977
	5	8.2380	8.2153
Woo et al. [8]	1	1.8439	1.8465
	2	2.0609	2.0505
	3	3.0595	3.2000
	4	3.2405	4.1801
	5	4.1708	4.4432

From Table 3, and Table 4, in the case of thin plate element (L/h=100) MITC3 elements converge to the reference solution in each skew angle. In fine mesh, mode 1, mesh 4 × 4 × 2 to mesh 128 × 128 × 2, the difference of skew plate θ = 30° to the reference solution is 1,585% - 84.808%, θ = 45° is 0,149% - 51.147%, θ = 60° is 0,059% - 34.823%, and θ = 75° to the reference solution is 0,001% - 27.188%.









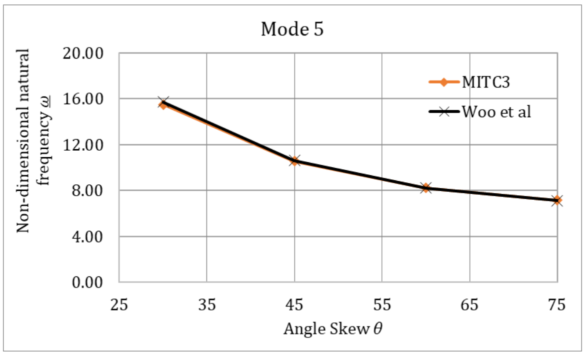


Figure 8. Natural Frequency of CFCF skew plate (L/h = 1000) according to skew angle.

Fig 8 depicts that increasing the skew angle θ caused a decrease in natural frequency value.

6. CONCLUSION

Free vibration analysis of skew plates in case of four skew angles, and two types of ratio plates of L/h =5 and L/h =1000 using MITC3 element, represents that there is the effect of skew angle and length to thickness ratios L/h, increasing skew angle caused decrease in natural frequency, and increasing the L/h caused increase in the natural frequency. Furthermore, MITC3 element convergence to reference solution and it concludes this element can be utilized to solve plate problem.

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